A distortion model for dead zone lattice vector quantization

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Abstract— **We propose a distortion model for a lattice vector quantization scheme with codebook shape adapted thresholding. We assume that the source (wavelet coefficients) have a laplacian distribution, which is a common hypothesis in the field of image compression. Combined with a bit-rate model recently proposed by the authors, this work permits to design an efficient bit allocation algorithm.**

Keywords— **Lattice Vector Quantization, vector dead zone, distorsion model, bit allocation, image compression.**

I. Introduction

Many works have been done [1][2][3][4][5][6][10] on Lattice Vector Quantization (LVQ) associated to Discrete Wavelet Transform (DWT) during the ten past years, in the field of image compression. The most of these works are based on an intraband approach whereas most efficient algorithms such that SPIHT [10] or JPEG2000 are based on the interband approach. However, interbands methods have some background specially concerning secure transmission. Moreover vector quantization benefits of the theoretical superiority from the point of view of the information theory.

It is also of interest to design an efficient LVQ based compression scheme: we have showed in [11] that the concept of vector dead zone yelds an improvement of visual quality compared to SPIHT and JPEG2000 algorithms (as it is shown in figure 1). Consequently, we have to design an efficient bit allocation algorithm. Here, we propose a theoretical distortion model for laplacian distributed sources which, combined to our rate model [11], allows to significantly simplify the bit allocation procedure by using analytical functions instead of processing data.

The paper is organized as follows. In section 2 we present the concept of vector dead zone. Section 3 is dedicated to the distortion model. Finally, section 4 presents some experimental results.

II. The Vector Dead Zone

The principle of our vector dead zone is following: by using a code book shape adapted thresholding, we can take into account significant blocks of wavelet coefficients. For a given rate, we can get a lower distortion by quantizing more precisely high energie vectors. Details on the lattice vector quantization scheme are given in [11], [4], [6].

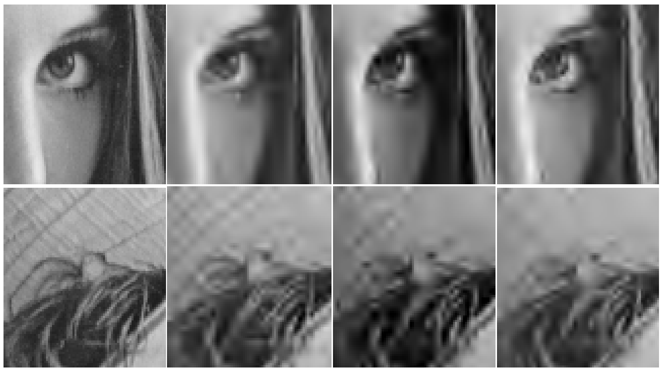

Fig. 1. original image of lena, DZLVQ, JPEG2000, SPIHT; Rate of 0.125 bpp (CR=1:64)

A. Definition

A pyramidal code book shape is well suited to laplacian distributed data for overload noise minimization purpose [5]. Thus we have proposed in [11] to design the codebook shape by including a pyramidal vector dead zone (see figure 2). It consists in replacing the voronoï of origin by an hyper-pyramid of radius R_{DZ} called dead zone (DZ). This new code book shape enables in one hand to minimize the overload distortion due to the source scaling within the code book and on the other hand to remove "non significant" source vectors, that is vectors belonging to the dead zone. Note, that the vector dead zone has a pyramidal shape too, since the source scaling also involves an overload distortion around the dead zone radius.

For a laplacian distributed source, we set:

$$DZ = \left\{ x \in R^n / \left\|x\right\|_1 = \sum_{i=1}^{n} |x_i| \leq R_{DZ} \right\} \tag{1}$$

With R_{DZ} the dead zone radius.

Let $\delta = \left\lceil \frac{R_{DZ}}{\gamma} \right\rceil$ be the radius of the first code book shell outside the dead zone. The whole lattice code book including the dead zone can be defined by:

$$C_{DZ} = \{ y \in Z^n / \delta \leq \left\|y\right\|_1 \leq R_T \} \cup \{0\}$$

where 0 is the null vector (see figure 2) and R_T the codebook truncation radius.

Let us explain now the quantization process itself.

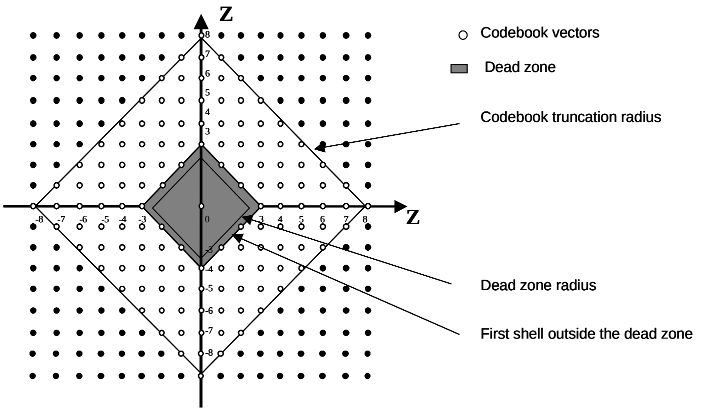

Fig. 2. The codebook with dead zone

B. The quantization process

In this paragraph we are going to describe how vectors are quantized according to the area.

In the folowing, we note $r_{DZ} = \frac{R_{DZ}}{\gamma}$ the dead zone radius on the scaled source.

The codebook is composed of three areas: the dead zone DZ, the overload zone OZ and the uniform zone UZ, as it is shown in figure 3. We can see on figure 3 an exemple of overload zone for $n = 2$.

In the dead zone, the vectors are quantized by 0.

The overload zone is defined by:

$$OZ = \{y \in Z^n / r_{DZ} \leq \|y\|_1 \leq \delta\} \tag{2}$$

OZ can be splitted in two sub-regions (see figure 3). Vectors belonging to NF are normally quantized. In F, vectors are not naturally quantized in the first shell outside the dead zone, but quantized inside the dead zone. Consequently these vectors must be rescaled. The corresponding factor is computed as follows:

Assume that X is a source vector belonging to F, γ the scaling factor, and x the scaled vector, we have $x = \frac{1}{\gamma} X = (x_1, ..., x_n)$.

Suppose that $\|Q(x)\| = E$, the norm of the quantized vector must increase of $\Delta = \delta - E$. Note that Δ is an integer. To quantize x on the shell δ, we have to determine a factor β such that:

$$\beta = \arg\min \{b \in]1, +\infty[/ \|Q(bx)\|_1 = \delta\} \tag{3}$$

We recall that $\delta = \left\lceil \frac{R_{DZ}}{\gamma} \right\rceil$ is the radius of the first code book shell outside the dead zone

We define β_i as: $\beta_i = \frac{x_i + sign(x_i)\frac{1}{2}}{x_i}$ which corresponds to the smallest factor permitting to increase by 1 the absolute value of the quantized component i. We sort $(\beta_i)_{i=1,...,n}$ in the ascending order, to obtain $\left(\overline{\beta}_j\right)_{j=1,...,n}$.

The factor verifying 3 is given by $\beta = \overline{\beta}_\Delta$, indeed we add 1 (in absolute value) to Δ quantized components, by this way the norm vector is equal to δ, it is quantized outside the dead zone.

Note that:

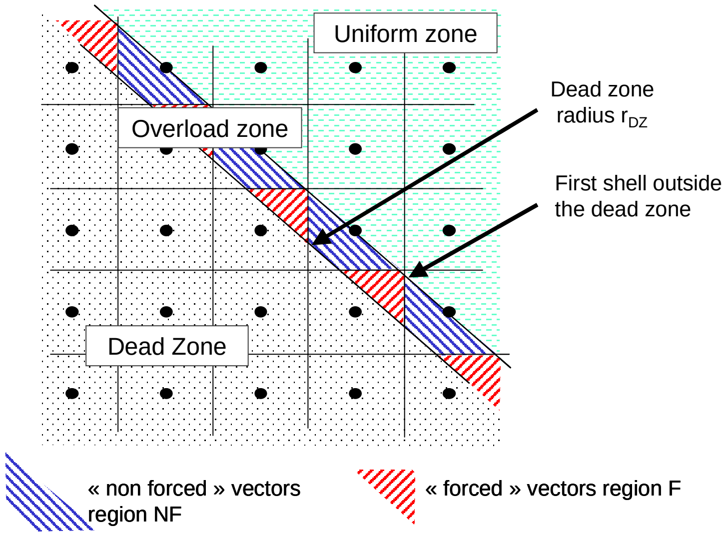

Fig. 3. the three code book regions

1. This operation is not an orthogonal projection, but the difference is neglictible because of the width of the OZ.
2. The coast of this operation is neglictible too as it is concerned only by few vectors.

In the third region (uniform zone) vectors are uniformly quantized.

III. The distortion model

A theoretical distortion model for DZLVQ presents two main advantages:

- first, it allows to proove from a theoritical point of view the interest of the vector dead zone approach,

- second, it permits to significantly simplify the bit allocation procedure by using analytical functions instead of processing data.

In the following, we are going to give the distortion in the three regions and finally, the global distortion.

A. Dead zone distortion

We suppose that $X = (x_1, ..., x_n) \in R^n$ is an i.i.d. random vector i.i.d, it is laplacian distributed with a standard deviation equals to σ and a null mean. The analytical expression of the dead zone distortion D_{DZ} in the case of a laplacian source is given by:

$$D_{DZ}(R_{DZ}) = n \left(J - 2\lambda e^{-\lambda R_{DZ}} R_{DZ}^3 \sum_{k=0}^{2} \frac{(\lambda R_{DZ})^k}{(k+3)!} \right) \tag{4}$$

with $J = -e^{-\lambda R_{DZ}} \left(R_{DZ}^2 + \frac{2}{\lambda} R_{DZ} + \frac{2}{\lambda^2}\right) + \frac{2}{\lambda^2}$, $\lambda = \frac{\sqrt{2}}{\sigma}$.

Proof: We not f_n the joint pdf of X and f the laplacian pdf in the scalar case. We have,

$$D_{DZ}(R_{DZ}) = \int_{\|X\|_1 < R_{DZ}} \|X\|_2^2 f_n(x_1, ..., x_n)\, dx_1...dx_n$$

$$= \int_{\|X\|_1 < R_{DZ}} \left(x_1^2 + ... + x_n^2\right) f(x_1)...f(x_n) dx_1...dx_n$$

as X is i.i.d. Thus,

$$D_{DZ}\left(R_{DZ}\right)=\sum_{i=1}^{n}\int_{\|X\|_1<R_{DZ}} x_i^2 f(x_1)...f(x_n)dx_1...dx_n$$
$$D_{DZ}\left(R_{DZ}\right)=n\int_{\|X\|_1<R_{DZ}} x_1^2 f(x_1)...f(x_n)dx_1...dx_n$$
$$=n\int_{|x|<R_{DZ}} x^2 f(x)\left[\int_{|x_2|+...+|x_n|<R_{DZ}-|x|} f(x_2)...f(x_n)dx_2...dx_n\right]$$

since we have,
$\{\|X\|_1 < R_{DZ}\} = \{|x_1| < R_{DZ}\} \cap A$
with $A = \{|x_2| + ... + |x_n| < R_{DZ} - |x_1|\}$

The bracketed integral is the repartition function of the radius law for a laplacian distribution F_{n-1} [5]. We obtain an analytical expression of F_n using successive per part integrations:

$$F_n\left(R\right)=\int_{\|X\|_1<R} f_n\left(x_1,...,x_n\right)dx_1...dx_n = 1-e^{-\lambda R}\sum_{k=0}^{n-1}\frac{\left(\lambda R\right)^k}{k!} \quad (5)$$

Consequently,
$$D_{DZ}\left(R_{DZ}\right)=n\int_{|x|<R_{DZ}} x^2 f(x)F_{n-1}\left(R_{DZ}-|x|\right)dx$$
$$=n\int_{|x|<R_{DZ}} x^2 f(x)\left[1-e^{-\lambda\left(R_{DZ}-|x|\right)}\sum_{k=0}^{n-1}\frac{\left(R_{DZ}-|x|\right)^k}{k!}\right]dx$$

By solving this integral we obtain the formula 4. ■

In the following we are going to focus on the distortion in the overload and uniform regions.

B. Distortion in both overload and uniform regions

As we can see in figure 3, some vectors belonging to the overload zone have to be rescaled to be quantized in the codebook. But the experimentation shows that the use of the vector dead zone allows to decrease significantly the scaling factor. Consequently:

1. the width w of this area depends on the scaling factor γ:

$$w = \gamma\delta - R_{DZ}, \quad (6)$$
$$\text{with } 0 < w \leq \gamma$$

w is small regarding to whole codebook, thus we consider the overload and the uniform zones as only one region.

2. We can do the high resolution hypothesis for vectors belonging to the uniform zone: we consider the quantization noise is uniformly distributed.

The distortion in the uniform zone is given by:
$$D_{UZ}\left(R_{DZ},\gamma\right)=\int_{\|X\|_1>R_{DZ}}\left\|X-\gamma Q\left(X\right)\right\|_2^2 f_n\left(x_1,...,x_n\right)dx_1...dx_n$$

In the uniform case, the well known [4] scalar distortion is equal to $\frac{\gamma^2}{12}$, consequently:
$$D_{UZ}\left(R_{DZ},\gamma\right)=\frac{n\gamma^2}{12}\int_{\|X\|_1>R_{DZ}} f_n\left(x_1,...,x_n\right)dx_1...dx_n$$

Finally we obtain the expression of distortion in the uniform region:

$$D_{UZ}\left(R_{DZ},\gamma\right)=\frac{n\gamma^2}{12}\left(\overline{F}_n\left(R_{DZ}\right)\right) \quad (7)$$

where $\overline{F}_n\left(R_{DZ}\right)=P\left(X\notin DZ\right)=1-F_n\left(R_{DZ}\right)$, F_n is given by 5.

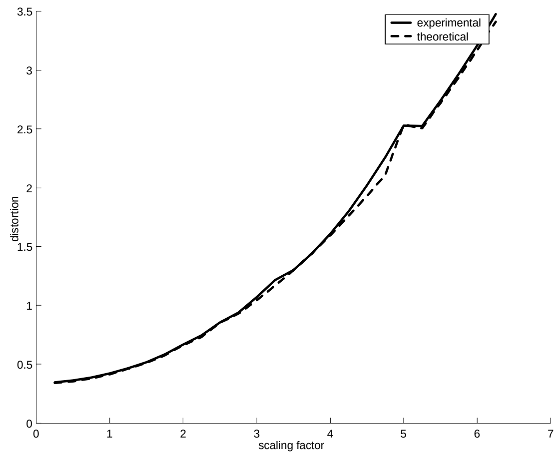

Fig. 4. Distortion function of the scaling factor; dead zone radius equals to 10; standard deviation of the laplacian source: 10

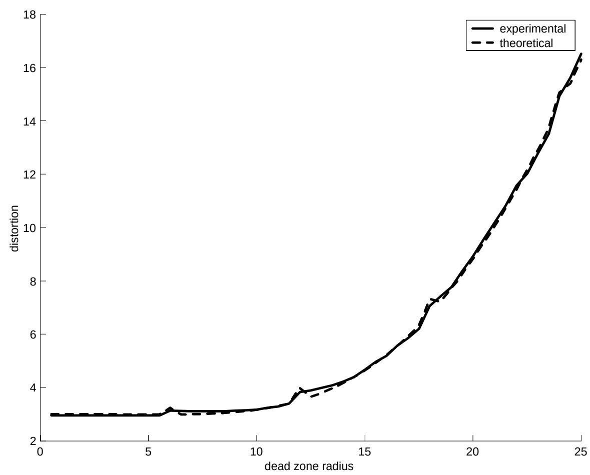

Fig. 5. Distortion function of the dead zone radius; scaling factor equals to 6; standard deviation of the laplacian source: 10

C. Global distortion

According to our scheme, the global distortion $D\left(R_{DZ},\gamma\right)$ per component can be expressed as:

$$D\left(R_{DZ},\gamma\right)=\frac{1}{n}\left[D_{DZ}\left(R_{DZ}\right)+D_{UZ}\left(R_{DZ},\gamma\right)\right] \quad (8)$$

In the next section we are going to verify the validity of our approximation.

IV. Theoretical vs. experimental results

Figures 4 and 5 show distortion as a function of γ and R_{DZ} respectively. The experimental results have been obtained using a synthetic laplacian distribution (standard deviation equals to 10 and average equals to 0), the size vectors is equal to 4. We can see on both figures that our model is accurate.

In figure 4, the dead zone radius has been fixed ($R_{DZ} = 10$), the model curve fits well the experimental distortion for scaling factors lower than 6, which represents here the limit of the high resolution hypothesis. In figure 5, γ is equal to 6, we can remark discontinuities on curves when $R_{DZ} = k\gamma$, $\gamma \in N$.

We have seen that our model was efficient when the high resolution hypothesis was available. It can also be associated to the rate model in order to determine the optimal parameter for a given rate.

V. Conclusion

In this paper we have presented an efficient distortion model for dead zone lattice vector quantization. We have shown that this model was accurate under the high resolution assumption. It allows, on one hand, to proove from a theoritical point of view the interest of the vector dead zone approach, and on the other hand it permits (combined to our rate model) to design an efficient bit allocation algorithm.

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